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Simultaneous heat and mass transfer in absorption of gases in turbulent liquid films

GERSHON GROSSMAN*
Energy Division, Oak Ridge National Laboratory, Oak Ridge, TN 37830, U.S.A.

and

MICHAEL T. HEATH
Computer Sciences Division, Oak Ridge National Laboratory, Oak Ridge, TN 37830, U.S.A.

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Abstract—This paper describes a theoretical analysis of the combined heat and mass transfer process taking place in the absorption of a gas or vapor in a turbulent liquid film. Eddy diffusivity correlations have been used to describe the transport near the wall and the interface and in the bulk of the film. The energy and diffusion equations are solved simultaneously to give the temperature and concentration variations in the film. Two cases of interest are considered: a constant-temperature and an adiabatic wall. The Nusselt and Sherwood numbers are expressed in terms of the operating parameters, from which heat and mass transfer coefficients are determined.

1. INTRODUCTION

THE ABSORPTION of gases and vapors in falling liquid films has received a great deal of attention in view of the frequent occurrence of this process in different types of gas–liquid contactors. Absorption generally involves simultaneous heat and mass transfer, but in many cases of importance, the mass transfer is dominant, and the effect of heat transfer is negligible. For this reason, most studies of the absorption process to date have considered only an isothermal exchange of mass [1–9]. In some important cases, however, the heat transfer accompanying the transfer of mass is significant and cannot be neglected. One typical example, from which the present study has originated, is in absorption heat pumps where absorbate vapor and absorbent liquid interact and mass is transferred specifically in order to produce a heat effect. Only little work has been done to date in modeling the process where the two transfer phenomena are strongly coupled [10–13].

In an earlier study [14], a model was developed to describe the simultaneous heat and mass transfer occurring in laminar falling films. The model, which followed other less elaborate ones [11–13], was based on the generally accepted assumptions of constant film properties, negligible changes in film thickness due to absorption, and thermodynamic equilibrium at the vapor–liquid interface. By solving simultaneously the energy and diffusion equations, the analysis yielded the temperature and concentration distributions in the film from which the heat and mass transfer coefficients could be calculated. The system variables, expressed in a normalized form, including the Sherwood and Nusselt numbers, were found to depend on the system's Peclet and Lewis numbers and on the dimensionless heat of absorption. Two cases of practical interest were considered. In one, the wall upon which the film flows was at a constant temperature; in the other, the wall was adiabatic.

The results have demonstrated, as well as quantified, the effect of the heat and the mass transfer on each other. Vapor absorbed at the liquid–gas interface releases its heat of absorption which raises the local temperature and, as a result, the solution vapor pressure. This reduces the film's capability to absorb more vapor, which in turn reduces further generation of heat. As heat is transferred away from the surface, the film becomes capable of absorbing additional vapor; as absorbate diffuses away from the surface, more heat can be generated.

In the present work, the model of ref. [14] has been extended to turbulent films, which are more common in practical applications. As in the case of the laminar films, earlier models of absorption under turbulent flow have been restricted to pure mass transfer at isothermal conditions. Important studies on this subject include those of Sandall and co-workers [7–9, 15, 16], Yih and Seagrave [17], and Mills and co-workers [18–21]. The purpose of the present study has been to model turbulent film absorption under the conditions of combined heat and mass transfer, to investigate the mutual effects of the two phenomena, and to find the dependence of the transfer coefficients on the system's operating parameters.

2. MODEL AND EQUATIONS

The system being analyzed is described schematically in Fig. 1. A film of liquid solution, composed of substances I (absorbent) and II (absorbate), flows down over an inclined plane. The film is in contact with stagnant vapor of substance II at constant pressure P_v. Substance I is contained only in the liquid phase;

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* On sabbatical from Faculty of Mechanical Engineering, Technion—Israel Institute of Technology, Haifa 3200, Israel.

NOMENCLATURE

C	concentration of absorbate in solution [mol m^{-3} solution]
C_e	equilibrium concentration of solution at temperature T_0 with vapor at pressure P_v [mol m^{-3} solution]
C_0	initial concentration of absorbate in solution [mol m^{-3} solution]
C_1, C_2	constants, equation (6c)
c_p	specific heat of liquid solution [J kg^{-1} °C^{-1}]
$\hat{c}$	numerical 'heat capacity', equation (17) [dimensionless]
D	diffusion coefficient of absorbate (substance II) in solution [m^2 s^{-1}]
D^*	generalized diffusion coefficient of absorbate, including turbulent effects [m^2 s^{-1}]
Fr	Froude number, $\bar{u}^2/\Delta g \sin\phi$ [dimensionless]
g	gravity [m s^{-2}]
$\bar{H}_a$	heat of absorption of substance II in solution [J mol^{-1}]
h_M	mass transfer coefficient from interface to bulk [m s^{-1}]
h_T	heat transfer coefficient from interface to bulk [W m^{-2} °C^{-1}]
h'_T	heat transfer coefficient from bulk to wall [W m^{-2} °C^{-1}]
K, K'	constants, equations (13a) and (13b) [dimensionless]
k	thermal conductivity of liquid solution [W m^{-1} °C^{-1}]
k^*	generalized thermal conductivity, including turbulent effects [W m^{-1} °C^{-1}]
i	enthalpy of liquid solution [J kg^{-1}]
Le	Lewis number, D/α [dimensionless]
Nu, Nu'	Nusselt number, $h_T\Delta/k$ and $h'_T\Delta/k$ [dimensionless]
Pr	Prandtl number, ν/α [dimensionless]
P_v	vapor pressure of absorbate (substance II) in the gas phase [Pa]
Re	Reynolds number, $4\bar{u}\Delta/\nu$ [dimensionless]
r_0, r_0^*, r	pipe-radius, normalized pipe radius, and radial coordinate in equation (13a)
Sc	Schmidt number, ν/D [dimensionless]
Sh	Sherwood number, $h_M\Delta/D$ [dimensionless]
T	temperature of solution [°C]
T_e	equilibrium temperature of solution at concentration C_0 with vapor at pressure P_v [°C]
T_0	initial temperature of solution [°C]
U	numerical variable representing either θ or γ, equation (17) [dimensionless]
u	flow velocity [m s^{-1}]
$\bar{u}$	average flow velocity [m s^{-1}]
v	normalized velocity, equation (7b) [dimensionless]
v_1	normalized velocity at the interface [dimensionless]
x	coordinate in direction of flow [m]
Y	numerical variable replacing η, equation (17) [dimensionless]
y	coordinate in direction perpendicular to flow [m]
y^*	normalized coordinate, $y\sqrt{(\tau_w/\rho)}/\nu$ [dimensionless]
z	similarity variable, equation (21a) [dimensionless].

Greek symbols

α	thermal diffusivity of liquid solution [m^2 s^{-1}]
α^*	generalized thermal diffusivity, including turbulent effects [m^2 s^{-1}]
γ	normalized concentration, equation (7c) [dimensionless]
$\gamma_i, \gamma_w, \bar{\gamma}$	normalized concentration at interface, wall, and bulk, respectively [dimensionless]
Δ	film thickness [m]
ε	eddy diffusivity [m^2 s^{-1}]
ζ	normalized coordinate in direction of flow, equation (7a) [dimensionless]
η	normalized coordinate perpendicular to flow, equation (7a) [dimensionless]
η_1	$(1-\eta)$, equation (18) [dimensionless]
θ	normalized temperature, equation (7c) [dimensionless]
$\theta_i, \theta_w, \bar{\theta}$	normalized temperature at interface, wall, and bulk, respectively [dimensionless]
λ	normalized heat of absorption, equation (7d) [dimensionless]
ν	kinematic viscosity of liquid [m^2 s^{-1}]
ρ	density of liquid [kg m^{-3}]
τ	shear stress [Pa]
τ_w	shear stress at the wall, $\rho\Delta g \sin\phi$ [Pa]
ϕ	inclination angle of film [rad].

substance II is transferred between the vapor and the liquid. At $x = 0$, the liquid film is at a uniform temperature T_0 and composition C_0 (moles of substance II per unit volume of solution) corresponding to an equilibrium vapor pressure lower than P_v. This results in a mass transfer process taking place at the liquid–vapor interface. The substance absorbed at the interface diffuses into the film; the heat generated in the absorption produces a simultaneous heat transfer process. Two cases are considered: in one, the wall is

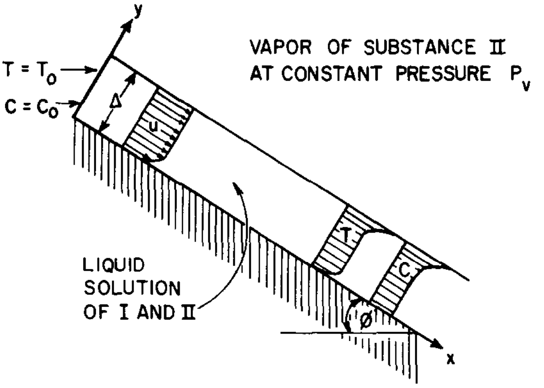

FIG. 1. Description of falling film of absorbent in contact with absorbate. Typical profiles of velocity, temperature, and concentration are shown.

kept at a constant temperature T_0; in the other, the wall is adiabatic.

The following assumptions have been made in formulating the model:

(1) The physical properties of the liquid solution are constant and independent of temperature and concentration.

(2) The mass of vapor absorbed per unit time is small compared to the mass flow rate of the liquid. Therefore, it is assumed that the latter is constant, and so are the film thickness and average flow velocity.

(3) Heat transfer in the vapor phase is negligible compared to that in the liquid phase.

(4) No shear forces are exerted on the liquid by the vapor.

(5) The flow is turbulent, with no thermally- or diffusion-induced momentum transfer.

(6) Vapor pressure equilibrium exists between the vapor and liquid at the interface.

Under these assumptions, the simultaneous heat and mass transfer in the system at steady-state is described by the diffusion and energy equations

$$u\frac{\partial C}{\partial x} = \frac{\partial}{\partial y}\left(D^*\frac{\partial C}{\partial y}\right) \tag{1}$$

$$u\frac{\partial(\rho i)}{\partial x} = \frac{\partial}{\partial y}\left(k^*\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial y}\left[D^*\frac{\partial C}{\partial y}\frac{\partial(\rho i)}{\partial C}\right] \tag{2}$$

where diffusion and heat conduction in the x-direction have been neglected with respect to those in the y-direction. Here u is the steady-state component of the turbulent flow velocity, i is the enthalpy of the solution, D^* and k^* are the generalized mass diffusivity and thermal conductivity, respectively, including the effect of turbulent eddies. The second term on the RHS of the energy equation, which does not exist in cases of pure heat transfer, expresses the flux of energy brought about by mass transfer.

A simplified form of equation (2) may be obtained by combining it with equation (1). Recognizing that $i = i(T, C)$, equation (2) may be rewritten as

$$\rho u\left(\frac{\partial i}{\partial T}\frac{\partial T}{\partial x} + \frac{\partial i}{\partial C}\frac{\partial C}{\partial x}\right) = \frac{\partial}{\partial y}\left(k^*\frac{\partial T}{\partial y}\right)$$
$$+\rho\frac{\partial i}{\partial C}\frac{\partial}{\partial y}\left(D^*\frac{\partial C}{\partial y}\right) + D^*\frac{\partial C}{\partial y}\rho\frac{\partial}{\partial y}\left(\frac{\partial i}{\partial C}\right)$$

multiplying equation (1) by $\rho(\partial i/\partial C)$ and subtracting, we find

$$\rho u\left(\frac{\partial i}{\partial T}\right)\frac{\partial T}{\partial x} = \frac{\partial}{\partial y}\left(k^*\frac{\partial T}{\partial y}\right) + D^*\frac{\partial C}{\partial y}\rho\frac{\partial}{\partial y}\left(\frac{\partial i}{\partial C}\right). \tag{3a}$$

The terms $(\partial i/\partial T)$ and $(\partial i/\partial C)$ express properties of the material in the liquid film. The former is the specific heat of the solution c_p; the latter, multiplied by the density ρ, is the partial molal enthalpy. Both quantities are reasonably constant for moderate changes of concentration and temperature. By virtue of assumption (1), the second term on the RHS of equation (3a) vanishes and the equation becomes

$$u\frac{\partial T}{\partial x} = \frac{\partial}{\partial y}\left(\alpha^*\frac{\partial T}{\partial y}\right) \tag{3b}$$

where $\alpha^* = k^*/\rho c_p$ is the generalized thermal diffusivity. Both α^* and D^* may be expressed as a sum of the corresponding molecular and turbulent (eddy) diffusivities, with the latter being dominant throughout most of the film, but vanishing at the wall and at the interface. Thus, equations (1) and (3b) become

$$u\frac{\partial C}{\partial x} = \frac{\partial}{\partial y}\left[(D+\varepsilon)\frac{\partial C}{\partial y}\right] \tag{4}$$

$$u\frac{\partial T}{\partial x} = \frac{\partial}{\partial y}\left[(\alpha+\varepsilon)\frac{\partial T}{\partial y}\right] \tag{5}$$

where ε is the eddy diffusivity (a function of y), assumed to be the same for heat and for mass transfer.

The boundary conditions applying to equations (4) and (5) are as follows. At the entrance plane, the initial temperatures and concentrations are given as

$$T = T_0 \quad \text{and} \quad C = C_0 \quad \text{at } x = 0. \tag{6a}$$

The wall is impermeable, and under the two cases considered, is either adiabatic or at a constant temperature equal to that of the entering solution. Hence

$$\left.\begin{array}{l}\dfrac{\partial C}{\partial y} = 0 \\ \text{or}\begin{cases} T = T_0 & \text{for constant temperature wall} \\ \partial T/\partial y = 0 & \text{for adiabatic wall}\end{cases}\end{array}\right\} \text{at } y = 0. \tag{6b}$$

The conditions at the interface require vapor pressure equilibrium between the vapor and liquid phases, and a match between the heat and the mass fluxes. This depends on the thermodynamic property relations of the particular fluid at hand. For a linear absorbent [14, 22], having a linear temperature–concentration characteristic and a constant heat of absorption, this condition becomes

$$\left.\begin{aligned} C &= C_1 T + C_2 \\ k(\partial T/\partial y) &= D(\partial C/\partial y)\bar{H}_a \end{aligned}\right\} \text{at } y = \Delta \qquad \text{(6c)}$$

where $\bar{H}_a$ is the heat of absorption, per mole of the vapor, in the liquid.

Typical shapes of velocity, temperature, and concentration profiles in the film are shown in Fig. 1. Before proceeding with the solution, it is convenient to rewrite the equations in a dimensionless form. Let us define the new variables

$$\zeta = \frac{4}{Re}\frac{x}{\Delta}; \quad \eta = \frac{y}{\Delta} \qquad \text{(7a)}$$

$$v = u/\bar{u} \qquad \text{(7b)}$$

$$\theta = \frac{T - T_0}{T_e - T_0}; \quad \gamma = \frac{C - C_0}{C_e - C_0} \qquad \text{(7c)}$$

$$\lambda = \frac{D(C_e - C_0)\bar{H}_a}{k(T_e - T_0)} \qquad \text{(7d)}$$

where T_e is the equilibrium temperature of the solution at concentration C_0 with the vapor, and C_e is the concentration of the solution at temperature T_0 in equilibrium with the vapor, both limiting values to those actually obtained in the simultaneous heat and mass transfer process [14]; $\bar{u}$ is the average flow velocity, equal to the mass flow rate per unit breadth, divided by the density and film thickness; and Re is the flow Reynolds number, defined on the basis of the hydraulic diameter of the film as

$$Re = \frac{4\bar{u}\Delta}{\nu}. \qquad \text{(8)}$$

Equations (4) and (5) with the new dimensionless variables become

$$v\frac{\partial \gamma}{\partial \zeta} = \frac{\partial}{\partial \eta}\left[\left(\frac{1}{Sc} + \frac{\varepsilon}{\nu}\right)\frac{\partial \gamma}{\partial \eta}\right] \qquad \text{(9)}$$

$$v\frac{\partial \theta}{\partial \zeta} = \frac{\partial}{\partial \eta}\left[\left(\frac{1}{Pr} + \frac{\varepsilon}{\nu}\right)\frac{\partial \theta}{\partial \eta}\right] \qquad \text{(10)}$$

where Pr and Sc are the Prandtl and Schmidt numbers, respectively. The boundary conditions now have the dimensionless form

$$\theta = 0 \quad \text{and} \quad \gamma = 0 \quad \text{at } \zeta = 0 \qquad \text{(11a)}$$

$$\left.\begin{array}{l} \dfrac{\partial \gamma}{\partial \eta} = 0 \\ \text{or} \begin{cases} \theta = 0 & \text{for constant temperature wall} \\ \partial\theta/\partial\eta = 0 & \text{for adiabatic wall} \end{cases} \end{array}\right\} \text{at } \eta = 0 \qquad \text{(11b)}$$

$$\theta + \gamma = 1 \quad \text{and} \quad \frac{\partial \theta}{\partial \eta} = \lambda\frac{\partial \gamma}{\partial \eta} \quad \text{at } \eta = 1. \qquad \text{(11c)}$$

At this point, the problem is completely defined in terms of the two second-order differential equations, equations (9) and (10), and boundary conditions (11a)–(11c) for the unknown distributions of θ and γ with ζ and η. It remains only to specify the eddy diffusivity ε in terms of the system's operating parameters and the η-coordinate.

It is not easy to use a single expression for describing the eddy diffusivity over the entire film thickness. A different behavior is exhibited in the laminar and buffer sublayers near the wall, in the turbulent core, and near the vapor–liquid interface [23]. The wall and the interface both have a damping effect on the eddies. In the turbulent core, the eddies produce considerable mixing which provides for low resistance to heat and mass transfer. Sandall [16], in his work on isothermal gas absorption in turbulent films, used a single expression which is normally valid only near the interface to model the eddy diffusivity in the entire film. Since his study was concerned with non-fully developed concentration profiles, the error incurred in doing so was small. Mills and Chung [18], in a study of pure heat transfer across a turbulent film, used two different expressions for the eddy diffusivity, one which is valid near the wall and the other near the interface. In doing so, the authors were able to predict successfully experimental results by Chun and Seban [24]. They did not find it necessary to accurately specify the eddy diffusivity in the middle region of the film, in view of the low thermal resistance there. In later experiments, however, Mills and co-workers [19, 20] produced data that showed the eddy diffusivity formula of ref. [18] to be inadequate, and have come up with a more appropriate ε expression [21], which was used to compare with the same experiments. Further, Seban and Faghri [25] have studied the required variation of ε in the turbulent core which was compared with Mills and Chung [18] and indicated that specification of the eddy diffusivity in this region yielded better agreement with experimental results.

In this study three different formulae were used to specify the eddy diffusivity in the three important regions of the film. The eddy diffusivity is assumed to be the same for heat, mass, and momentum transfer. Near the wall, we chose the expression suggested by Van Driest [26]

$$\frac{\varepsilon}{\nu} = \frac{1}{2}\{-1 + [1 + 0.64y^{*2}(1 - \exp(-y^*/26))^2]^{1/2}\} \qquad \text{(12a)}$$

where $y^* = y\sqrt{(\tau_w/\rho)}/\nu$, and $\tau_w = \rho\Delta g \sin\phi$ is the shear stress at the wall. Equation (12a) may be rewritten in terms of the dimensionless variables used in our equations as

$$\frac{\varepsilon}{\nu} = \frac{1}{2}\{-1 + [1 + 0.04(Re^2\,\eta^2/Fr) \times (1 - \exp(-Re\,\eta/104Fr))^2]^{1/2}\} \qquad \text{(12b)}$$

where Fr is the Froude number. Equation (12b) was used in the sublayers region, $0 \leqslant y^* < 30$ or $0 \leqslant \eta < 120\sqrt{Fr}/Re$. In the middle region of the film we used Riechardt's formula [27], proposed for the turbulent core in a pipe of radius r_0

$$\frac{\varepsilon}{\nu}=\frac{K}{6}r_0^*\left[1-\left(\frac{r}{r_0}\right)^2\right]\left[1+2\left(\frac{r}{r_0}\right)^2\right]. \qquad (13a)$$

Equation (13a) may be adapted for the present problem by replacing r_0 by Δ and r by $(\Delta-y)$

$$\frac{\varepsilon}{\nu}=K'\frac{Re}{\sqrt{Fr}}(2\eta-\eta^2)(3-4\eta+2\eta^2) \qquad (13b)$$

where K' is determined so as to intersect equation (13b) with equation (12b) at $\eta = 120\sqrt{Fr}/Re$.

The specification of ε in the interface region is more complex. The expressions selected by Mills and co-workers [18, 21] were based on the notion that the eddy damping in the film near the interface is affected largely by surface tension, as suggested earlier by Levich [28]. Other investigators [29, 30] have stated that the major mechanism for eddy damping is viscosity, basing their claim on a theoretical model and data from more than ten sources. In an attempt to reconcile the differences, it was found that the major discrepancy between Won and Mills' latest results [20] and those of others, is that the former indicate a variation of the mass transfer coefficient with D to a power dependent on surface tension, whereas most other investigators obtain a $D^{1/2}$ variation. We further found that if we used surface tension values in Won and Mills' formula [20] that reduced the power of D to $\frac{1}{2}$, their results are in good agreement with the correlations of other data, developed recently by Yih and Chen [30]. If we consider in the present model absorbents satisfying this requirement, it is possible to use the reduced version of Won and Mills' formula [20] for the mass transfer coefficient to obtain an eddy diffusivity expression in the interface region, which is in good agreement with most presently available data. This was done following the model by Lamourelle and Sandall [7] to give

$$\frac{\varepsilon}{\nu}=3.153\times10^{-7}Fr^{-2/3}\,Re^{2.758}\left(\frac{y}{\Delta}\right)^2. \qquad (14)$$

Equation (14) was used between $\eta = 1$ and the point of intersection with equation (13b).

The expressions for the eddy diffusivity may be used to calculate the velocity profile. Noting that the shear stress varies linearly with y and is zero at the interface, we can write

$$\tau=\tau_w(1-\eta)=\frac{\rho}{\Delta}(\nu+\varepsilon)\frac{\partial u}{\partial\eta}, \qquad (15)$$

and hence

$$v=\frac{u}{\bar{u}}=\frac{Re}{4Fr}\int_0^\eta\frac{(1-\eta)}{(1+\varepsilon/\nu)}\,\mathrm{d}\eta \qquad (16)$$

which satisfies $v = 0$ at $\eta = 0$.

The dimensionless flow parameters of a given film, Re and Fr, are not independent. To find the relation between them, a numerical algorithm was set up where a value of the Froude number is guessed for a given Reynolds number, and used to calculate v by substituting equations (12b), (13b), and (14) in equation (16). The resulting v must conform to the condition

$$\int_0^1 v\,\mathrm{d}\eta = 1$$

and the guessed value of Fr is adjusted accordingly until this requirement is satisfied.

Figure 2(a) describes velocity profiles for several turbulent Reynolds numbers. The corresponding Froude numbers are listed in parentheses. As expected, the velocity profiles become flatter as the Reynolds number increases, and higher Froude numbers are

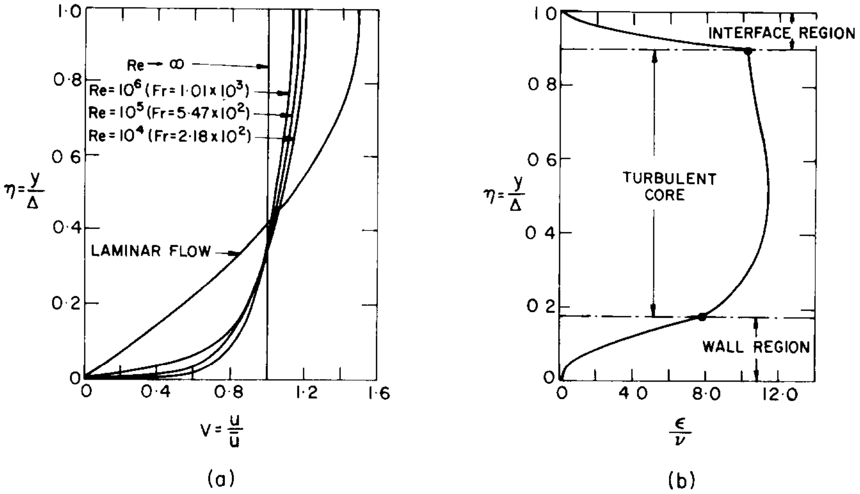

FIG. 2. Hydrodynamics of the falling film: (a) velocity profiles for different values of the Reynolds number (and corresponding Froude number); (b) eddy diffusivity profile for $Re = 10^4$, showing the three different regions, as described by equations (12b), (13b), and (14).

required to produce them. Figure 2(b) shows a typical profile of eddy diffusivity describing the three different regions.

3. SOLUTION

The numerical technique used to solve the partial differential equations (9) and (10) was similar to the one described in ref. [14] and based on the so-called 'method of lines' or 'semi-discretization' [31]. The ζ-η plane of the film was divided into thin strips by means of lines parallel to the ζ-axis. This discretization of the η-coordinate made it possible to express the derivatives with respect to η in each of the equations in a finite-difference form. Thus, a first-order ordinary differential equation, in ζ alone, was obtained along each line. Such an equation could be readily solved by means of an available ODE integrator using boundary condition (11a). The integrator selects automatically the required step in ζ and varies it as necessary while the integration proceeds.

Instead of carrying out a simultaneous solution of the two equations for θ and γ, it was found more convenient to define a single variable U, to be solved for from one equation

$$\frac{\partial U}{\partial \zeta} = \frac{1}{\hat{c}} \frac{\partial}{\partial Y}\left(\hat{k} \frac{\partial U}{\partial Y}\right) \tag{17}$$

in the domain $0 \leqslant \zeta < \infty$, $-1 \leqslant Y \leqslant 1$. In the half-domain $-1 \leqslant Y < 0$, equation (17) replaces equation (10) with $Y \equiv \eta - 1$, $U \equiv \theta$, $\hat{c} \equiv v\,Pr$, and $\hat{k} \equiv (1 + Pr\,\varepsilon/v)$. In the half-domain $0 < Y \leqslant 1$, equation (17) replaces equation (9) with $Y \equiv 1 - \eta$, $U \equiv 1 - \gamma$, $\hat{c} \equiv \lambda v\,Sc$, and $\hat{k} \equiv \lambda(1 + Sc\;\varepsilon/v)$. We recognize equation (17) as the heat conduction equation with a variable 'thermal conductivity' $\hat{k}$ and 'heat capacity' $\hat{c}$. Boundary condition (11a) transforms in the new formulation into $U = 0$ for $-1 \leqslant Y < 0$ and $U = 1$ for $0 < Y \leqslant -1$ at $\zeta = 0$; boundary condition (11b) transforms into $U = 0$ or $\partial U/\partial Y = 0$ at $Y = -1$, and $\partial U/\partial Y = 0$ at $Y = 1$; boundary condition (11c) is satisfied automatically. We have thus converted our problem, which involved the simultaneous solution of two coupled equations, into one where we seek a solution to a single equation with the above boundary conditions in a domain twice as wide as the film.

Due to the boundary-layer nature of the problem, it was necessary to begin the solution with a very fine mesh at small ζ, where very strong gradients exist, particularly near the interface. In the interest of saving computer time and storage, the mesh points were initially concentrated in the boundary-layer region, and the mesh size was gradually expanded as the integration proceeded with ζ. Even with the very fine mesh, some difficulty was encountered in trying to apply the method very close to the point $\zeta = 0$, $\eta = 1$, due to the discontinuity in the temperature and concentration between the interface and the entrance plane at this point. To overcome this problem, an analytical solution applicable close to the singular point was developed, which made it possible to calculate the values of the variables at some finite distance away from the point and begin the numerical solution from there.

The analytical solution is similar to the one used in ref. [14] and earlier in ref. [13] for laminar flow. By defining a new variable

$$\eta_1 = 1 - \eta \tag{18}$$

and recognizing that for $\eta_1 \ll 1$, the velocity v is practically constant with η_1, and ε/v is much smaller than $1/Pr$ and $1/Sc$, we can rewrite equations (9) and (10) near the singular point as

$$v_1 \frac{\partial \gamma}{\partial \zeta} = \frac{1}{Sc} \frac{\partial^2 \gamma}{\partial \eta_1^2} \tag{19}$$

$$v_1 \frac{\partial \theta}{\partial \zeta} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial \eta_1^2} \tag{20}$$

where v_1 is the value of v at $\eta = 1$. Boundary conditions (11b) and (11c) now apply at $\eta_1 \to \infty$ and $\eta_1 = 0$, respectively. It is then possible to find a similarity variable, combining both ζ and η_1, for each of the equations and convert them from partial to ordinary ones. Using the common similarity technique, equation (19) becomes

$$\frac{\mathrm{d}^2 \gamma}{\mathrm{d}z^2} = -2z \frac{\mathrm{d}\gamma}{\mathrm{d}z} \tag{21a}$$

where $z = \eta_1 \sqrt{(v_1\,Sc/4\zeta)}$. Equation (21a) may be integrated twice to give

$$\gamma = k_1 \operatorname{erf}(z) + k_2 = k_1 \operatorname{erf}\left(\eta_1 \sqrt{\left(\frac{v_1\,Sc}{4\zeta}\right)}\right) + k_2. \tag{21b}$$

In a similar manner we find from equation (20)

$$\theta = k_3 \operatorname{erf}\left(\eta_1 \sqrt{\left(\frac{v_1\,Pr}{4\zeta}\right)}\right) + k_4 \tag{22}$$

where k_1, k_2, k_3, and k_4 are constants of integration. Applying boundary condition (11a) yields $k_1 = -k_2$ and $k_3 = -k_4$ since $\operatorname{erf}(\infty) = 1$. Boundary condition (11b) is satisfied automatically for both the adiabatic and constant temperature walls. Boundary condition (11c) then yields $k_1 + k_3 = -1$ and $\lambda k_1 \sqrt{Sc} = k_3 \sqrt{Pr}$ from which all the constants of integration can finally be determined. We, thus, obtain the following expressions for the dimensionless temperature and concentrations, in terms of the original variables

$$\theta = \frac{\lambda}{\lambda + \sqrt{Le}}\left[1 - \operatorname{erf} \sqrt{\left(\frac{v_1\,Pr(1-\eta)^2}{4\zeta}\right)}\right] \tag{23}$$

$$\gamma = \frac{\sqrt{Le}}{\lambda + \sqrt{Le}}\left[1 - \operatorname{erf} \sqrt{\left(\frac{v_1\,Sc(1-\eta)^2}{4\zeta}\right)}\right] \tag{24}$$

where $Le = Pr/Sc$ is the Lewis number. Note that equations (23) and (24) are valid for small ζ, for both the adiabatic and constant temperature wall cases. This is to be expected since the effect of the wall cannot be felt

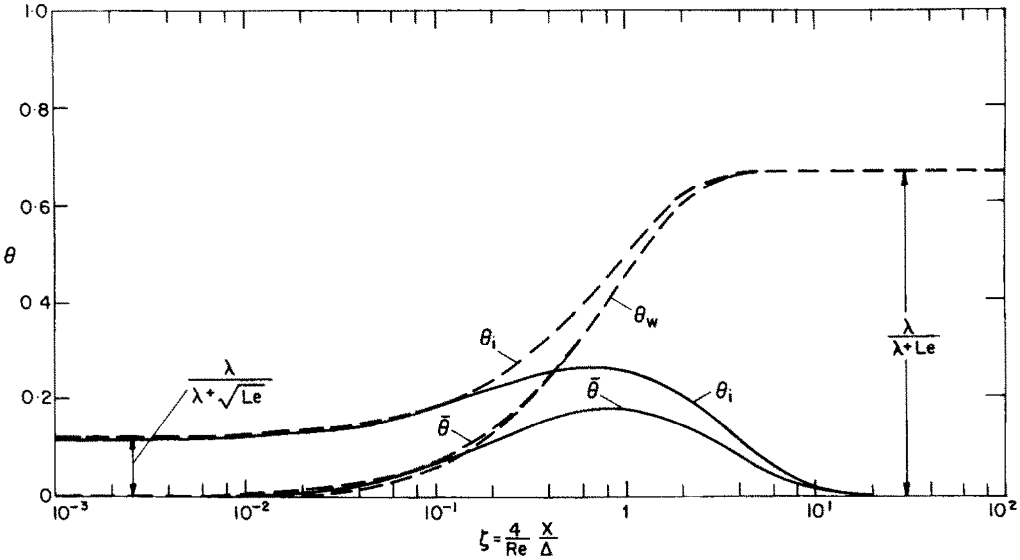

FIG. 3. Dimensionless wall, liquid bulk and interface temperatures as functions of the normalized length ζ for $Re = 10^4$, $Pr = 10.0$, $Sc = 2000.0$, and $\lambda = 0.01$. Broken lines, adiabatic wall; solid lines, constant-temperature wall.

until the boundary layer developing from the interface has had enough distance to fill the entire film thickness.

4. RESULTS AND DISCUSSION

Figures 3 and 4 describe the general behavior of the temperature and concentration in the system as they vary with the normalized length ζ, for a typical set of values of the parameters Re, Pr, Sc, and λ. Plots are shown for θ and γ at the wall (θ_w, γ_w), in the liquid bulk ($\bar{\theta}$, $\bar{\gamma}$), and at the liquid–vapor interface (θ_i, γ_i). The solid lines describe the results for the constant-temperature wall and the broken lines for the adiabatic wall. This notation will be maintained throughout this section.

The behavior exhibited here is similar, qualitatively, to that observed in the combined heat and mass transfer process in laminar films [14]. The changes with ζ in the present case are, however, considerably more rapid due to the improved transport in turbulent flow. Initially, for very small ζ, the behavior is the same for the adiabatic and constant-temperature wall cases. The

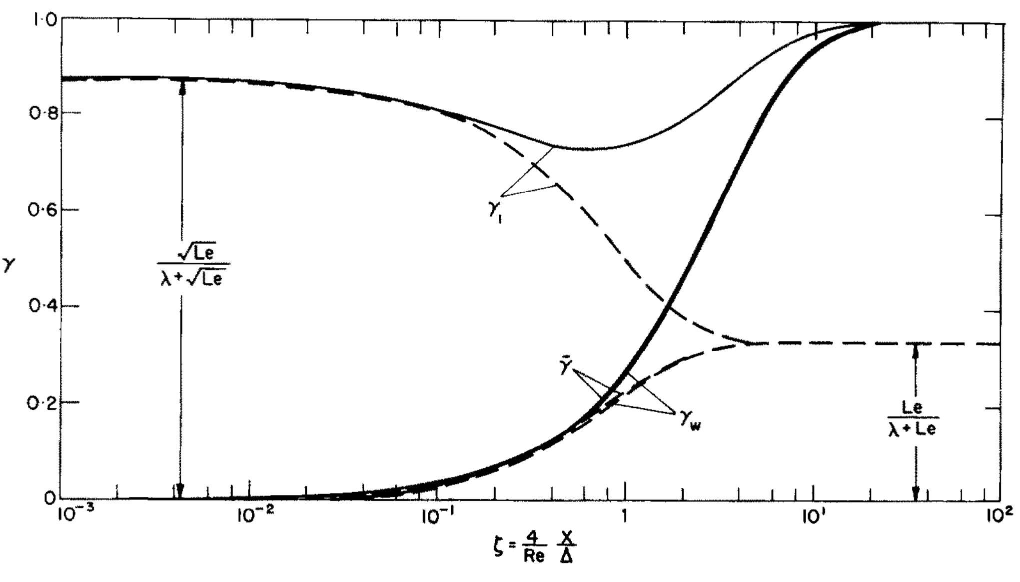

FIG. 4. Dimensionless wall, liquid bulk and interface concentrations as functions of the normalized length ζ for $Re = 10^4$, $Pr = 10.0$, $Sc = 2000.0$, and $\lambda = 0.01$. Broken lines, adiabatic wall; solid lines, constant-temperature wall.

liquid at the interface reaches thermodynamic equilibrium with the vapor upon contact at $\zeta = 0$, but some distance is required for the effect to diffuse into the film and be noticed at the wall. Consequently, θ_w and γ_w remain essentially zero for small ζ while θ_i and γ_i remain almost constant at their initial values reached at $\zeta = 0$. These values are $\lambda/(\lambda+\sqrt{Le})$ and $Le/(\lambda+\sqrt{Le})$, respectively, as found from the similarity solution for small ζ, equations (23) and (24).

As ζ increases, a thermal and a concentration boundary layer develop and grow in thickness until they fill the entire depth of the film. For the adiabatic wall case, the wall, bulk, and interface temperatures increase monotonically toward a common final value. This steady increase is due to the fact that the heat of absorption is not removed from the system. With the constant-temperature wall, the interface temperature increases somewhat, following the trend of the adiabatic case, and is followed by the bulk temperature as heat is transferred from the interface into the film. Then, both temperatures decrease toward zero as heat is taken out of the system through the wall. The interfacial concentration in both cases follows a trend opposite to that of the interfacial temperature, since $\gamma_i = 1-\theta_i$ [equations (11c)]. The bulk concentration increases in both cases toward a final value equal to that of γ_i. It is interesting to note that in the adiabatic wall case $\bar{\gamma}$ increases with ζ while γ_i decreases.

The asymptotic values of the dimensionless temperature and concentration in the constant-temperature wall case are 0 and 1, respectively. In the adiabatic wall case, these values may be found from the diffusion and energy equations, equations (9) and (10). Integrating over the film thickness and using the boundary conditions (11b) yields

$$\frac{d}{d\zeta}\int_0^1 v\gamma \, d\eta = \frac{1}{Sc}\left.\frac{\partial \gamma}{\partial \eta}\right|_{\eta=1} \tag{25a}$$

$$\frac{d}{d\zeta}\int_0^1 v\theta \, d\eta = \frac{1}{Pr}\left.\frac{\partial \theta}{\partial \eta}\right|_{\eta=1} \tag{25b}$$

with the aid of boundary conditions (11c), these two equations may be combined and the result integrated with respect to ζ to give

$$\int_0^1 v\theta \, d\eta = \frac{\lambda}{Le}\int_0^1 v\lambda \, d\eta. \tag{26}$$

The integrals on the LHS and RHS of equation (26) represent the bulk temperature and concentration, $\bar{\theta}$ and $\bar{\gamma}$, respectively. Since at $\zeta \to \infty$, the bulk, wall, and interface quantities are all equal to each other, equation (26) expresses the ratio between the asymptotic temperature and concentration as λ/Le. This ratio, combined with the condition $\theta+\gamma = 1$, yields $\theta = \lambda/(Le+\lambda)$ and $\gamma = Le/(\lambda+Le)$ at $\zeta \to \infty$.

Typical temperature and concentration profiles across the film for increasing values of ζ illustrate the gradual penetration of the interface effects into the film. The profiles differ markedly from those in the laminar case [14] in that they are flat throughout the middle region of the film, owing to the turbulent mixing there. Sharp temperature and concentration gradients are observed in the interface region, which is where most of the resistance to heat and mass transfer lies. In the constant-temperature wall case, additional resistance to heat transfer in the wall region is manifested by a temperature gradient there.

Figure 5 describes the bulk concentration $\bar{\gamma}$ as a

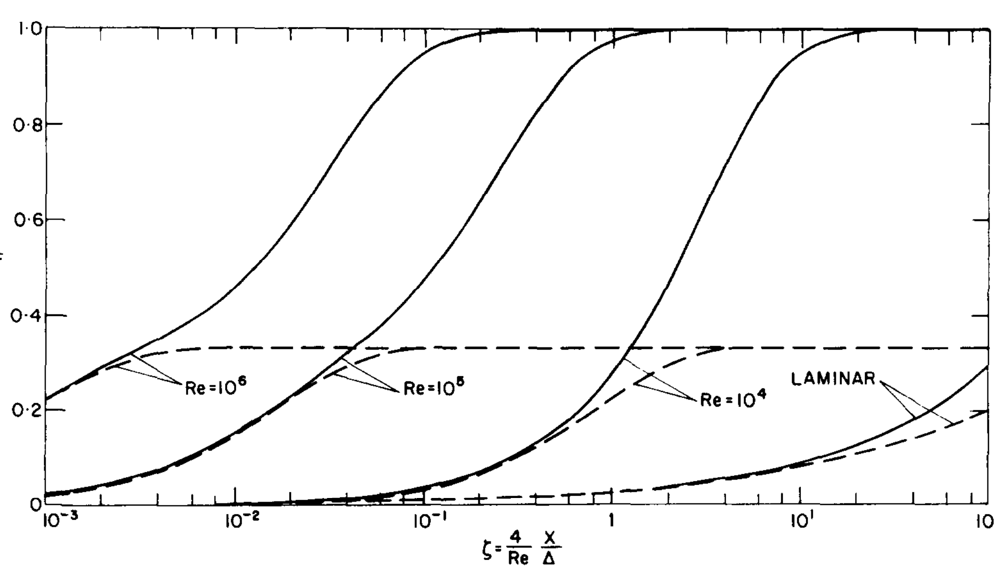

FIG. 5. Dimensionless bulk concentration as a function of the normalized length ζ for $\lambda = 0.01$, $Pt = 10.0$, $Sc = 2000.0$, and different values of Re. Broken lines, adiabatic wall; solid lines, constant-temperature wall.

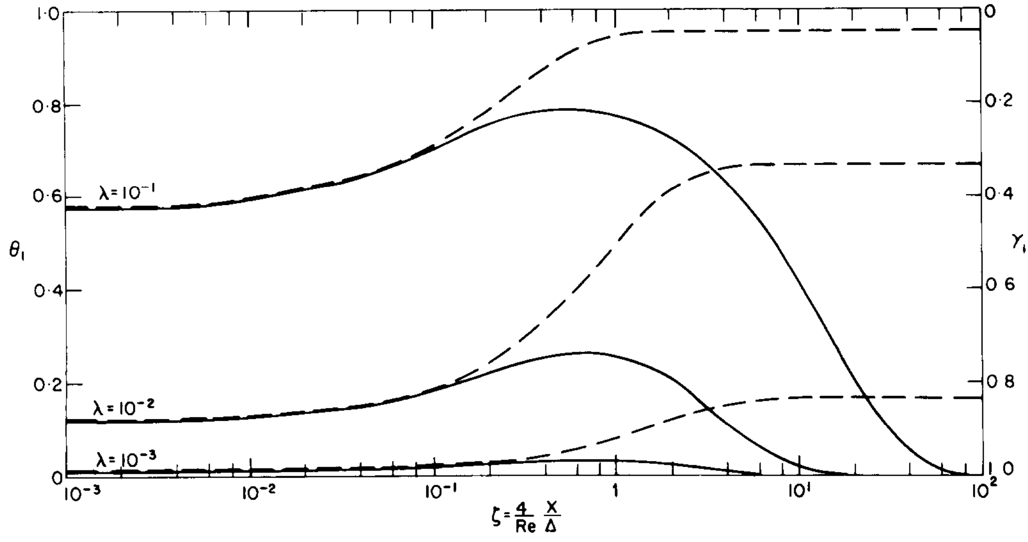

FIG. 6. Dimensionless interface temperature and concentration as functions of the normalized length ζ for $Re = 10^4$, $Pr = 10.0$, $Sc = 2000.0$, and different values of λ. Broken lines, adiabatic wall; solid lines, constant-temperature wall.

function of the dimensionless distance ζ for three typical values of the Reynolds number in the turbulent range, and for laminar flow. The other characteristic parameters of the system are maintained fixed, and, hence, the asymptotic concentration is the same in all cases. As expected, the enhanced transport associated with higher Reynolds numbers makes it possible for the concentration (and temperature) to reach the asymptotic value at a smaller ζ.

The effect of the heat of absorption λ is shown in Fig. 6, describing the temperature and concentration at the interface as functions of ζ. Each curve represents either θ_i or γ_i when read on the corresponding scale, since $\theta_i + \gamma_i = 1$. We observe that the initial ($\zeta = 0$) value of the interface temperature increases with λ, and that of the interface concentration decreases, according to the similarity solution formulas shown in Figs. 3 and 4. The same is true for the asymptotic values. At the limit of $\lambda = 0$ (negligible heat of absorption), $\theta_i = 0$ and $\gamma_i = 1$ for all ζ. Under this condition, there is no difference between the adiabatic and constant-temperature wall cases.

The effect of the Prandtl and Schmidt numbers is described in Fig. 7, which shows the interface

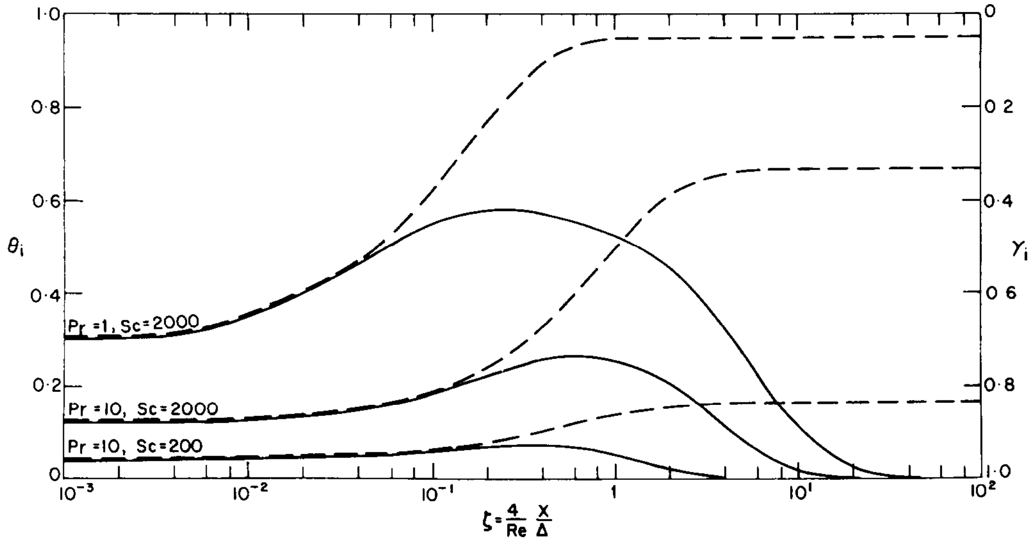

FIG. 7. Dimensionless interface temperature and concentration as functions of the normalized length ζ for $Re = 10^4$, $\lambda = 0.01$, and different values of Pr and Sc. Broken lines, adiabatic wall; solid lines, constant-temperature wall.

temperature and concentration in a manner similar to that of Fig. 6. The main influence of *Pr* and *Sc* is on the initial and asymptotic values of θ and γ, where only the ratio Pr/Sc appears in the form of the Lewis number.

The literature uses several different definitions of the transfer coefficients in processes where heat and mass transfer are coupled. It seems best to use a definition based on the quantity difference which constitute the driving force for the transfer phenomena. The coefficient of local mass transfer from the interface to the bulk of the liquid is defined through the Sherwood number as

$$Sh = \frac{h_M \Delta}{D} = \frac{[\partial\gamma/\partial\eta]_{\eta=1}}{(\gamma_i - \bar{\gamma})}. \tag{27}$$

The coefficient of local heat transfer from the interface to the bulk of the liquid is defined through the Nusselt number as

$$Nu = \frac{h_T \Delta}{k} = \frac{[\partial\theta/\partial\eta]_{\eta=1}}{(\theta_i - \bar{\theta})}. \tag{28}$$

In the constant-temperature wall case there is also a need to consider the heat transfer coefficient from the bulk of the fluid to the wall. Hence

$$Nu' = \frac{h'_T \Delta}{k} = \frac{[\partial\theta/\partial\eta]_{\eta=0}}{\bar{\theta}}. \tag{29}$$

Figure 8 describes the Sherwood number as a function of the normalized length ζ for different values of the system's parameters. Under the investigated range, there is no noticeable difference between the adiabatic and constant-temperature wall cases. *Sh* is very large for small ζ and decreases toward an asymptotic value as ζ increases. We note, first, that for fixed λ, *Pr*, and *Sc*, the Sherwood number increases with *Re* which provides for better transport, especially in the critical interface zone. For fixed *Re*, *Sh* increases with the Schmidt number, but is not sensitive to the Prandtl number. The effect of λ is small and limited to the region of small ζ, before the asymptotic value has been reached.

Figure 9 describes a plot similar to that of Fig. 8 for the Nusselt number. *Nu* exhibits a behavior very similar to that of *Sh*, with the exception that the former is sensitive to *Pr* and insensitive to *Sc*. *Nu'*, relating to the heat transfer at the wall, was found to be affected mainly by *Re* and *Pr*, and insensitive to λ and *Sc*.

An important difference between the laminar and turbulent films is the relative sensitivity in the former, and insensitivity in the latter, of the Sherwood number to the heat of absorption λ. This indicates that the mass transfer coefficient in the combined heat and mass transfer process is essentially the same as in isothermal mass transfer. The coupling between the two transfer phenomena is manifested primarily through the driving forces, the temperature and concentration differences between the film interface and bulk.

5. CONCLUSIONS

A model was developed for analysis of the combined heat and mass transfer processes in absorption of vapor in turbulent liquid films. Eddy diffusivity correlations were used to describe the transport in the wall, bulk, and interface regions of the film. The energy and diffusion equations were solved simultaneously with an equilibrium boundary condition at the vapor–liquid interface. Two cases of practical importance were considered—a constant-temperature wall and an adiabatic wall.

The results of the solution describe the variation of

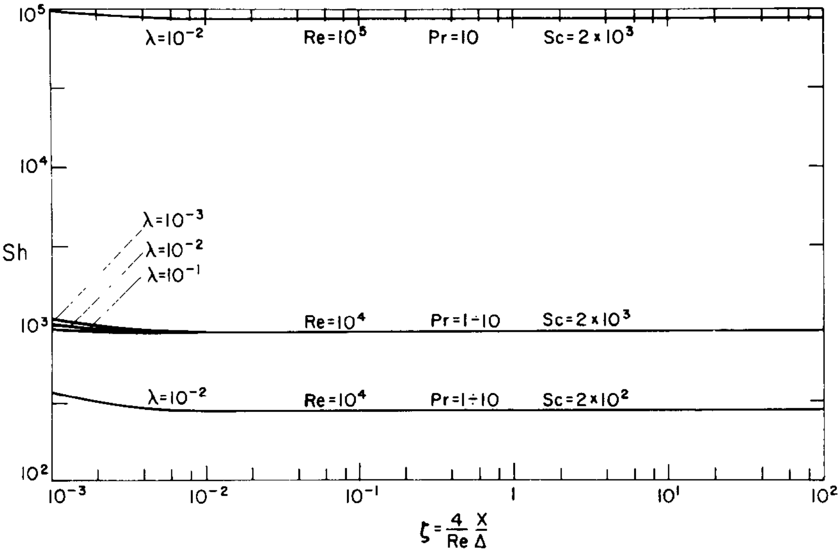

FIG. 8. Local Sherwood number as a function of the normalized length ζ for different values of the system's parameters.

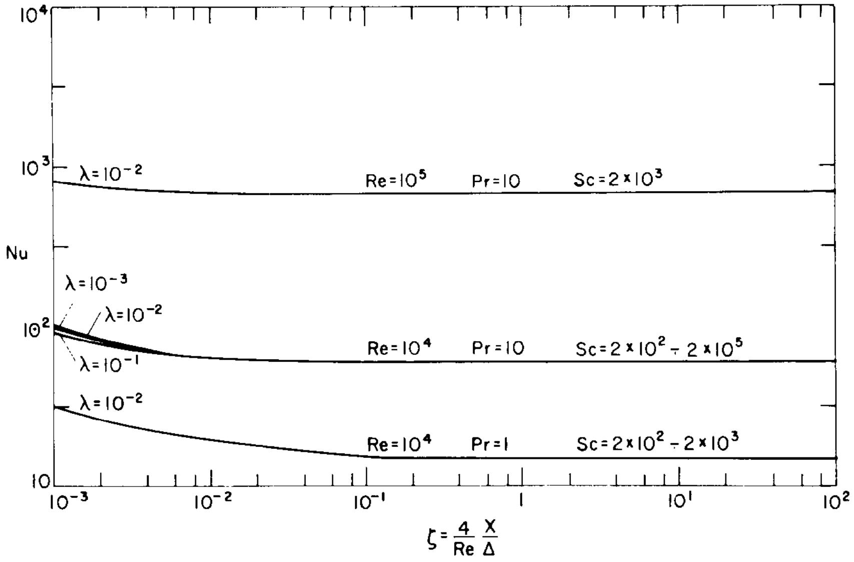

FIG. 9. Local Nusselt number as a function of the normalized length ζ. Broken lines, adiabatic wall; solid lines, constant-temperature wall.

the temperatures and concentrations within the film. These quantities in their normalized, dimensionless form depend on four characteristic parameters of the system: the Reynolds, Prandtl, and Schmidt numbers, and the dimensionless heat of absorption λ. In the constant-temperature wall case, the dimensionless temperature and concentration reach asymptotic values of 0 and 1, respectively. In the adiabatic wall case the asymptotic temperature and concentration are $\lambda/(\lambda + Le)$ and $Le/(\lambda + Le)$, respectively.

Heat and mass transfer coefficients for the system were calculated. The Sherwood number for mass transfer from the vapor–liquid interface to the bulk of the film increases with Re and Sc, and is insensitive to Pr. The Nusselt number for heat transfer from the interface to the bulk increases with Re and Pr, and is insensitive to Sc. Both Sh and Nu reach asymptotic values after a fairly short distance in the direction of flow, and are little influenced by the heat of absorption λ. The coupling between the heat and mass transfer processes is mainly through the interdependent driving forces—the temperature and concentration differences.

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TRANSFERT SIMULTANE DE CHALEUR ET DE MASSE DANS L'ABSORPTION DE GAZ DANS DES FILMS LIQUIDES TURBULENTS

Résumé—On décrit une analyse théorique des mécanismes de transferts combinés de chaleur et de masse dans l'absorption d'un gaz ou d'une vapeur par un film liquide turbulent. On utilise des expressions de diffusivités turbulentes pour décrire le transfert près de la paroi et de l'interface et dans le coeur du film. Les équations d'énergie et de diffusion sont résolues simultanément pour obtenir les variations de température et de concentration dans le film. On considère deux cas intéressants: température pariétale constante et paroi adiabatique. Les nombres de Nusselt et de Sherwood sont exprimés en fonction des paramètres opératoires et on détermine les coefficients de transfert à partir d'eux.

GLEICHZEITIGER WÄRME- UND STOFFÜBERGANG BEI DER ABSORPTION VON GASEN IN TURBULENTE FLÜSSIGKEITSFILME

Zusammenfassung—Diese Arbeit enthält eine theoretische Untersuchung des kombinierten Wärme- und Stoffübergangs bei der Absorption von Gas oder Dampf in einen turbulenten Flüssigkeitsfilm. Zur Beschreibung der Transportvorgänge in der Nähe der Wand, der Grenzfläche und im Kern des Films werden Korrelationen für den Scheindiffusionskoeffizienten verwendet. Die Energie- und Stoffübergangsgleichungen werden zur Ermittlung der Temperatur- und Konzentrationsänderungen im Film gleichzeitig gelöst. Zwei Fälle werden betrachtet: Absorption bei konstanter Temperatur und Absorption bei adiabater Wand. Die Nusselt- und Sherwood-Zahlen werden mit Hilfe der Betriebsparameter gebildet, die den Wärme- und Stoffübergangskoeffizienten zugrundeliegen.

ОДНОВРЕМЕННЫЙ ТЕПЛО-И МАССОПЕРЕНОС ПРИ ПОГЛОЩЕНИИ ГАЗОВ ПЛЕНКАМИ ТУРБУЛЕНТНОЙ ЖИДКОСТИ

Аннотация—Представлен теоретический анализ процесса связанного тепло-и массопереноса при поглощении газа или пара пленкой турбулентной жидкости. Для описания переноса вблизи стенки, на границе и в объеме пленки используются соотношения турбулентного переноса, основанные на концепции вихревой температуропроводности и диффузии. Уравнения энергии и диффузии решаются совместно с целью определения изменений температуры и концентрации в пленке. Рассматриваются два случая: стенка с постоянной температурой и адиабатическая стенка. Числа Нуссельта и Шервуда выражаются через параметры, определяющие коэффициенты тепло-и массопереноса.